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Nonlinearc Aeroelastic Analysis of an Airfoil Using CFD-Based Indicial Approach

Dong-Hyun Kim*
GyeongSang National University, Gyeongnam, Jinju 660-701, South Korea
In Lee†
Korea Advanced Institute of Science and Technology, Daejon 305-701, South Korea
Piergiovanni Marzocca‡
Clarkson University, Potsdam, New York 13699-5725
Liviu Librescu§
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061-0219
and
Stephen Schober¶
Clarkson University, Potsdam, New York 13699-5725

Nomenclature

a	=	dimensionless elastic axis position measured from the midchord, positive aft
b	=	half-chord length
$C_{l\alpha}$	=	lift-curve slope
C_L; C_M	=	lift and moment coefficients
h	=	plunging displacement about the pitch axis
I_α	=	$\int_c x^2 \rho_s \, \mathrm{d}x$, cross section mass moment of inertia about its elastic axis, per unit span
K_h, K_α	=	spring stiffness in plunge and pitch directions, respectively
k_b	=	reduced frequency ($\equiv \omega b/U_\infty$)
m	=	airfoil mass per unit span
q	=	pitch rate about the reference axis ($\equiv (2b/U_\infty)\dot{\alpha}$)
r_α	=	dimensionless radius of gyration about elastic axis (EA)
S_α	=	$m x_{\mathrm{cg}} = \int_x x \rho_s \, \mathrm{d}x$, static moment per unit span
t, τ	=	time variable and its dimensionless counterpart ($\equiv tU_\infty/b$), respectively
U_∞	=	free-stream speed
w	=	generic functions
x_α	=	dimensionless static unbalance of the airfoil about its elastic axis; CG-EA offset
α	=	twist angle about the pitch axis
ρ_∞	=	Air density
σ	=	Dummy time variable
ϕ^c_α; $\phi^c_{M\alpha}$; ϕ^c_q; ϕ^c_{Mq}	=	compressible lift and moment indicial functions associated with the pitch angle α and the pitch rate q
ω, k	=	Circular and reduced frequencies ($k = k_b \equiv \omega b/U_\infty$)
$()'$	=	$\mathrm{d}()/\mathrm{d}\tau$, differentiation with respect to the dimensionless time τ

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*Assistant Professor, School of Mechanical and Aerospace Engineering (with Research Center for Aircraft Part Technology). Member AIAA.

†Professor, Department of Aerospace Engineering. Senior Member AIAA.

‡Assistant Professor, Department of Mechanical and Aerospace Engineering. Member AIAA.

§Professor, Department of Engineering Science and Mechanics.

¶Graduate Student, Department of Mechanical and Aeronautical Engineering. Student Member AIAA.

Introduction

LINEAR and nonlinear aerodynamic indicial functions concepts constitutes a powerful method for determining unsteady aerodynamic loads in various flight speed regimes and implicitly investigating the aeroelastic phenomena such as flutter instability and dynamic response in various flight-speed regimes. The present paper focuses on the use of the concept of nonlinear indicial aerodynamic functions in conjunction with computational fluid dynamics (CFD) for determination of the unsteady aerodynamic loads and their use in aeroelastic analyses. The indicial functions produced by an input that instantaneously evolves from an initial value to a final value enable us to represent the aerodynamic loads acting on lifting surfaces in response to arbitrary external excitation. In particular, for a two-degree-of-freedom airfoil, the aeroelastic motion can be obtained by the combination of pitch and plunge displacements. For the compressible flow, assuming a linearized model, the unsteady lift and aerodynamic moment in time and frequency domains are obtained by using the compressible counterparts of indicial Wagner and Theodorsen functions, respectively,[1–3] whereas, for the transonic flow field, the unsteady lift and aerodynamic moment are derived in terms of the nonlinear indicial functions developed in Ref. 4. In Ref. 4, an indicial formulation of unsteady aerodynamics was presented highlighting the importance of the aerodynamic indicial function concept. For the nonlinear indicial theory, it is assumed that the indicial responses of a nonlinear system can be parameterized based on the instantaneous motion state.[5] These parameterized indicial responses form a kernel that can be determined in principle from computation, experiments, or analysis. New algorithms are applied to identify the system's kernel of indicial and aeroelastic responses. As is well known, for the linear approach, if the indicial aerodynamic responses can be determined, then they can be used to determine the aerodynamic lift and moment in time or frequency domains.[4–6] As a result of shock waves and/or shock-induced flow separation in the transonic flow regime, the incorporation of aerodynamic nonlinearities becomes an essential requirement for the study of related aeroelastic phenomena. Consequently, in this flight speed regime a nonlinear indicial approach has to be used.

This note represents an extension of Ref. 4 for the transonic flight speed regime, where the indicial unsteady aerodynamic functions were used for the incompressible and compressible flight speed regimes. In the present study, transonic aeroelastic responses are computed based on a new nonlinear indicial function approach. For a two-degree-of-freedom airfoil, time-domain unsteady aerodynamic forces, dynamic response, and the flutter boundary are determined via the new nonlinear indicial function method and compared with those obtained by direct CFD.

Theoretical Background

The aeroelastic model considered here is a two-degree-of-freedom system, free to rotate in the x–y plane and free to translate in the vertical direction. The plunging deflection is denoted by h, which is positive in the downward direction at the elastic axis (EA); α is the pitch angle about the elastic axis, which is positive in nose-up rotation. The elastic axis is located at a distance ab from midchord, whereas the mass center is located at a distance $x_\alpha b$ from the elastic axis. Both distances are positive when measured toward the trailing edge of the airfoil. Note that the pitching axis may be offset from the center of mass of the airfoil toward the midchord, leading to coupling between the pitching and plunging degrees of freedom. The airfoil is assumed to be constrained to move along the vertical y-axis and to rotate about the elastic axis. The dimensionless aeroelastic governing equations for a two-degree-of-freedom airfoil model are as follows:

$$\xi''(\tau) + x_\alpha \alpha''(\tau) + 2\zeta_h(\bar{\omega}/\bar{U})\xi'(\tau) + (\bar{\omega}/\bar{U})^2\xi(\tau) = -l_a(\tau;\xi,\alpha;\xi',\alpha';\xi'',\alpha'') - l_g(\tau) - l_b(\tau) \tag{1}$$

$$\left(x_\alpha \big/ r_\alpha^2\right)\xi''(\tau) + \alpha''(\tau) + 2\zeta_\alpha(1/\bar{U})\alpha'(\tau) + (1/\bar{U})^2\alpha(\tau) = m_a(\tau;\xi,\alpha;\xi',\alpha';\xi'',\alpha'') + m_g(\tau) \tag{2}$$

where

$$l_a(\tau) = (1/\pi\mu)c_l(\tau), \qquad m_a(\tau) = \left(2\big/\pi\mu r_\alpha^2\right)c_m(\tau)$$

$$\bar{U} = U_\infty/b\omega_\alpha, \qquad \bar{\omega} = \omega_h/\omega_\alpha$$

$$\omega_h = \sqrt{k_h/m}, \qquad \omega_\alpha = \sqrt{k_\alpha/I_\alpha}, \qquad \mu = m\big/\pi\rho_\infty b^2$$

$$x_\alpha = S_\alpha/mb, \qquad r_\alpha = \sqrt{I_\alpha\big/mb^2}$$

Here, $\xi(\tau) = h(\tau)/b$ and $\alpha(\tau)$ are the normalized plunging and pitching displacements at the elastic axis, respectively. On the right-hand sides of Eqs. (1) and (2), l_a and m_a stand for the nondimensional unsteady aerodynamic lift and moment, respectively. The variables l_g and m_g denote the nondimensional gust load and moment, while l_b denotes the nondimensional blast load. In the present indicial approach, the unsteady lift and moment in the time domain are computed through inverse Laplace transforms of those expressed in the frequency domain. Details on the determination of the unsteady aerodynamic load based on linear and nonlinear indicial function approach can be found in Refs. 4 and 7.

The nonlinear nature of the mixed flow (subsonic and supersonic) in the transonic flight speed range, characterized by large-amplitude shock-wave motion, can significantly invalidate the linear assumption. In addition, changes in the angle of attack will produce changes in the indicial function. This is a major difference between the linear and nonlinear models, and thus a proper nonlinear indicial model has to be used in the transonic flow region. The nonlinear representation of unsteady lift $\bar{L}_a^c$ and aerodynamic moment $\bar{M}_a^c$ in the compressible flight-speed regime evaluated at the leading edge ($x = -b$), for arbitrary plunging and pitching about the leading edge, can be expressed as

$$\begin{aligned}\bar{L}_a^c = {} & -C_{l\alpha}b\rho_\infty U_\infty^2 \bar{\phi}_\alpha^c[\tilde{w}(0);\tau]\left(\alpha(0) - \frac{h'(0)}{b}\right) \\ & + 2C_{l\alpha}b\rho_\infty U_\infty^2 \alpha'(0)\bar{\phi}_q^c[\tilde{w}(0);\tau] \\ & - C_{l\alpha}b\rho_\infty U_\infty^2 \int_0^\tau \left\{\bar{\phi}_\alpha^c[\tilde{w}(\varsigma);\tau,\sigma]\left(\alpha'(\sigma) + \frac{h''(\sigma)}{b}\right)\right. \\ & \left. - 2\bar{\phi}_q^c[\tilde{w}(\varsigma);\tau,\sigma]\alpha''(\sigma)\right\} \mathrm{d}\sigma \end{aligned} \tag{3a}$$

$$\begin{aligned}\bar{M}_a^c = {} & -C_{l\alpha}b\rho_\infty U_\infty^2 \bar{\phi}_{M\alpha}^c[\tilde{w}(0);\tau]\left(\alpha(0) - \frac{h'(0)}{b}\right) \\ & + 2C_{l\alpha}b\rho_\infty U_\infty^2 \alpha'(0)\bar{\phi}_{Mq}^c[\tilde{w}(0);\tau] \\ & - C_{l\alpha}b\rho_\infty U_\infty^2 \int_0^\tau \left\{\bar{\phi}_{M\alpha}^c[\tilde{w}(\varsigma);\tau,\sigma]\left(\alpha'(\sigma) + \frac{h''(\sigma)}{b}\right)\right. \\ & \left. - 2\bar{\phi}_{Mq}^c[\tilde{w}(\varsigma);\tau,\sigma]\alpha''(\sigma)\right\} \mathrm{d}\sigma \end{aligned} \tag{3b}$$

where the new nonlinear indicial functions can be defined as

$$\bar{\phi}_\alpha^c[\tilde{w}(\varsigma);\tau,\sigma] = \lim_{\Delta\tilde{w}\to 0} \frac{\Delta\bar{C}_{L\alpha}^c(\tau)}{\Delta\tilde{w}} \tag{4a}$$

$$\bar{\phi}_{M\alpha}^c[\tilde{w}(\varsigma);\tau,\sigma] = \lim_{\Delta\tilde{w}\to 0} \frac{\Delta\bar{C}_{M\alpha}^c(\tau)}{\Delta\tilde{w}} \tag{4b}$$

$$\bar{\phi}_q^c[\tilde{w}(\varsigma);\tau,\sigma] = \lim_{\Delta\tilde{w}\to 0} \frac{\Delta\bar{C}_{Lq}^c(\tau)}{\Delta\tilde{w}} \tag{4c}$$

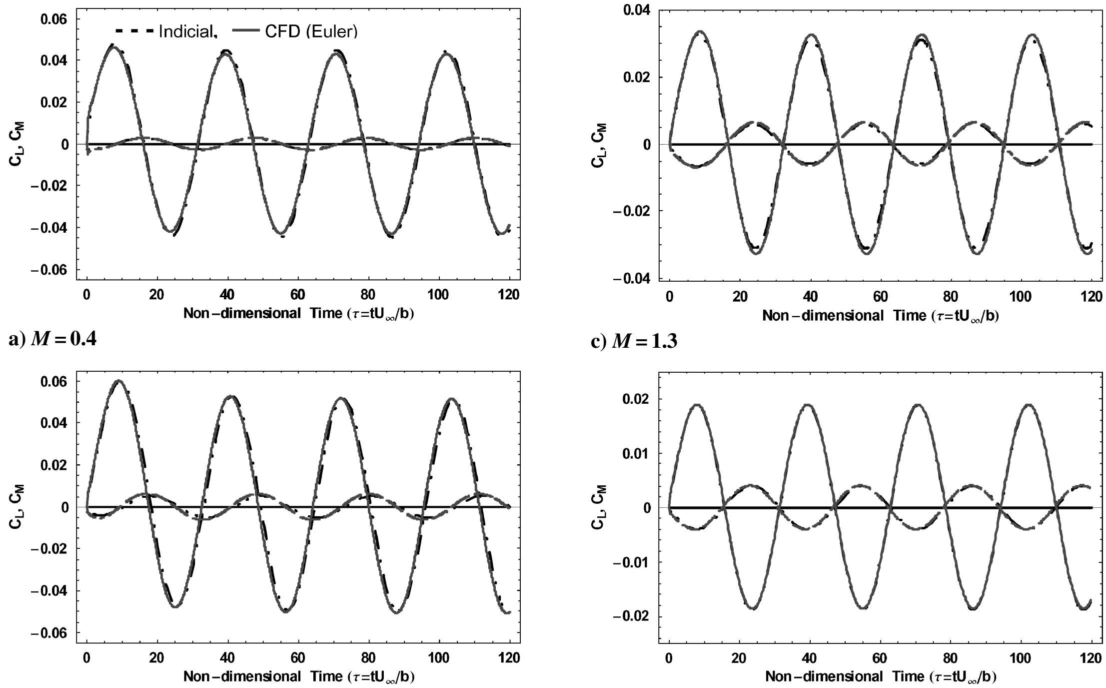
$$\bar{\phi}_{Mq}^c[\tilde{w}(\varsigma);\tau,\sigma] = \lim_{\Delta\tilde{w}\to 0} \frac{\Delta\bar{C}_{Mq}^c(\tau)}{\Delta\tilde{w}} \tag{4d}$$

Equations (3) and (4) can be adapted to case of arbitrary plunging and pitching about the point located at the distance ab rearward to the reference midchord.

Results and Discussion

The unsteady aerodynamic lift and moment have been obtained from linear and nonlinear indicial response functions used in conjunction with the generalized Duhamel superposition principle. These results are compared with direct CFD calculations on airfoils undergoing oscillations in plunge and pitch in the transonic flight speed regime. Generally, normal shock waves in unsteady flow on an airfoil with dynamic motions play an important role in determining nonlinear aerodynamic characteristics. The unsteady flow quantities are obtained from the two-dimensional time-dependent Euler equations. For the unsteady aerodynamic/aeroelastic response analyses, an arbitrary Lagrangean–Eulerian formulation for the Euler equations is used to calculate the flow flux in a scheme with moving boundaries. To validate unsteady aerodynamic calculations based on the present nonlinear indicial function (NIF) method, a NACA 0006 airfoil with the pitching motion $\alpha(\tau) = 0.5^\circ \sin(k_b\tau)$ at 25% chord axis and freestream Mach numbers of 0.4, 0.8, 1.3, and 2.0 considered here. Both the linear/nonlinear indicial approach and two different unsteady CFD (Euler) codes with dynamic meshing algorithm[8,9] are used. Comparisons of unsteady aerodynamic lift and moment coefficients are presented in Fig. 1. It is shown that in the compressible flow-speed range, the present NIF method provides very good agreements with the results obtained from the direct CFD method.

A two-degree-of-freedom typical section aeroelastic model with a NACA 0006 airfoil is considered for this computational demonstration. Whereas the stiffness of the structural model is assumed to be linear, aerodynamic nonlinearities generally occur in the transonic flow field. The structural parameters for the typical section model considered are $a = -0.5$, $\mathcal{X}_\alpha = 0.3$, $r_\alpha = 0.5$, $\mu = 20$, and $\bar{\omega} = 0.2$–1.8. Unless otherwise stated, a step-loading initial condition is applied. One of the major practical aspects of the present NIF method is the fact that it is very fast and provides accurate predictions of the flutter. In addition, this methodology can have important applications in nonlinear aeroservoelasticity in transonic and low-supersonic flow regimes. From point of view of computational time, the present NIF method is extremely efficient especially for problems requiring extended long-time responses such as typical limit-cycle-oscillation phenomena. Moreover, based on the Laplace transform, the NIF method can be used to obtain the time-history of the response. Although the direct CFD method can accurately handle strong nonlinear flows with unsteady normal shock waves, the computational time closely depends on the grid size and number



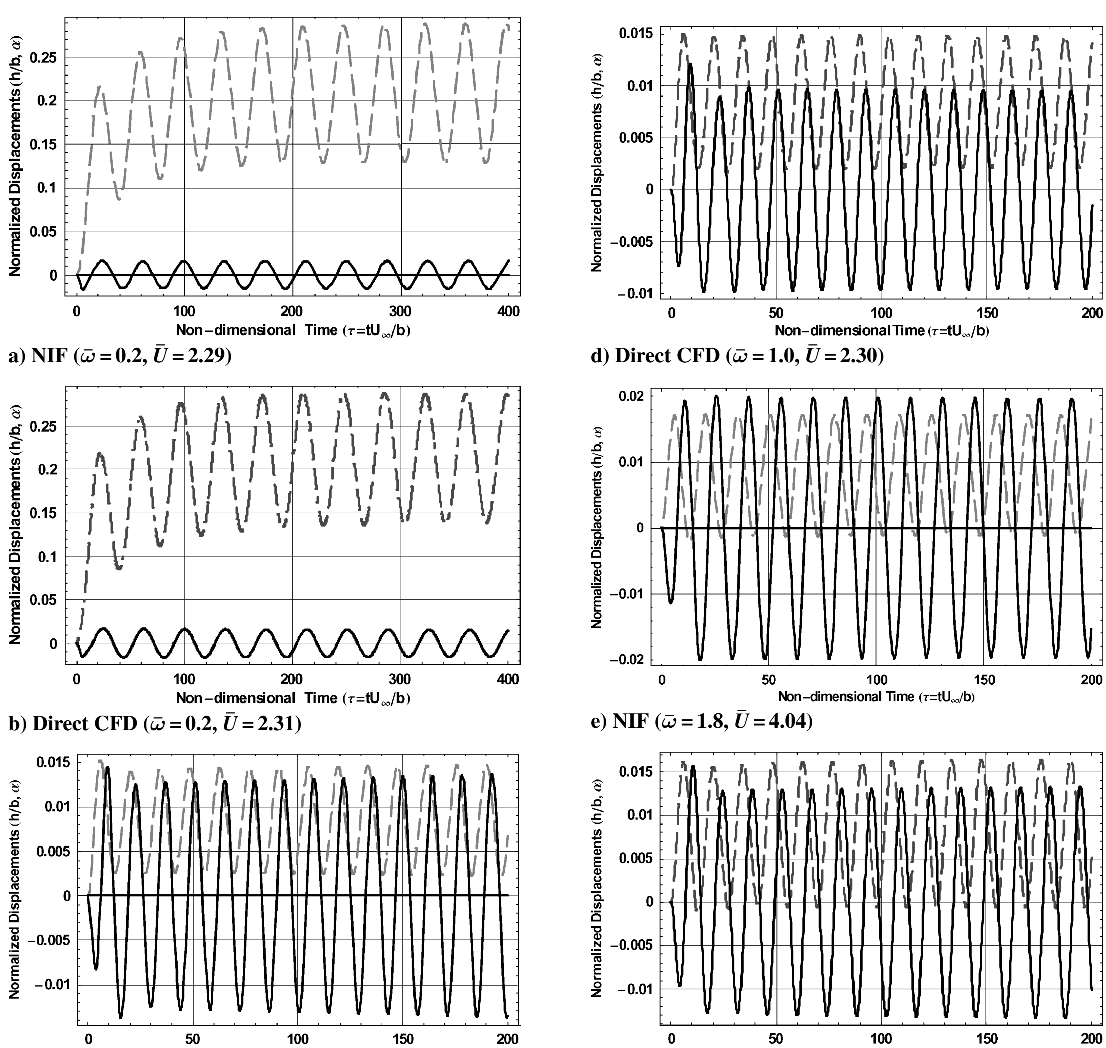
a) $M = 0.4$

b) $M = 0.8$

c) $M = 1.3$

d) $M = 2.0$

Fig. 1 Comparison of unsteady lift and moment coefficients during pitching oscillations at different Mach numbers (NACA 0006, $k_b = 0.2$).



a) NIF ($\bar{\omega} = 0.2$, $\bar{U} = 2.29$)

b) Direct CFD ($\bar{\omega} = 0.2$, $\bar{U} = 2.31$)

c) NIF ($\bar{\omega} = 1.0$, $\bar{U} = 2.31$)

d) Direct CFD ($\bar{\omega} = 1.0$, $\bar{U} = 2.30$)

e) NIF ($\bar{\omega} = 1.8$, $\bar{U} = 4.04$)

f) Direct CFD ($\bar{\omega} = 1.8$, $\bar{U} = 3.99$)

Fig. 2 Comparison of aeroelastic responses on the verge of flutter instability for various frequency ratios at $M = 0.8$.

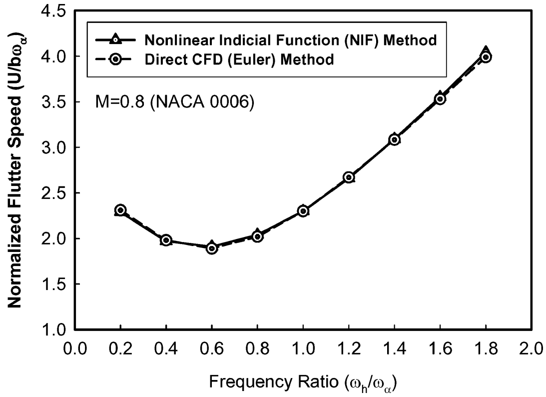


Fig. 3 Comparison of flutter boundary with the variation of frequency ratio at $M = 0.8$ (KP-B model).

of iterations required for a maximum response time. In addition, the CFD method generally requires small time-steps or subiterations to keep numerical stability and accuracy. Figure 2 shows the comparison of transonic aeroelastic responses for selected frequency ratios at $M = 0.8$. In the CFD computations, the nondimensional time-step size of 0.01–0.02 is used for all cases. In the figure, the dashed line indicates the plunge displacement and the solid line indicates the pitch displacement. Here, we can see similar neutral responses at nearly the same non-dimensional flutter speeds for the models with different frequency ratios. To show the practical applications for aeroelastic design problems, a comparison of the flutter boundary is presented in Fig. 3. In this case, the frequency ratio is a major design parameter. As revealed in the figure, the flutter velocity attains a minimum in the vicinity of the frequency ratio $\bar{\omega} = 0.6$. Also, the present NIF method predicts nearly the same flutter boundary that is determined by the direct CFD method. It is worth noting that the frequency ratio vs normalized flutter speed can be also dependent on freestream Mach number. This means that the trend of the curve can change according to the Mach number in the transonic and supersonic flow-speed regimes.

Conclusions

In this paper, a nonlinear indicial approach in conjunction with computational fluid dynamics (CFD) has been developed to analyze the transonic aeroelastic response and determine the flutter boundary of general two-degree-of-freedom airfoil systems. The use of the nonlinear indicial function was shown to be successful in addressing these issues in the transonic flow regime. For a two-degree-of-freedom typical section model, the predictions based on the present nonlinear indicial function show very good agreements with results by the direct CFD method. It seems that the present approach can have a strong potential for practical application in problems related to nonlinear aeroelastic phenomena such as transonic aeroservoelasticity and limit cycle oscillations.

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References

[1] Marzocca, P., Librescu, L., and Chiocchia, G., "Aeroelastic Response of a 2-D Lifting Surfaces to Gust and Arbitrary Explosive Loading Signatures," *International Journal of Impact Engineering*, Vol. 25, No. 1, 2001, pp. 41–65.

[2] Marzocca, P., Librescu, L., and Chiocchia, G., "Aeroelastic Response of a 2-D Airfoil in Compressible Flight Speed Regimes Exposed to Blast Loadings," *Aerospace Science and Technology*, Vol. 6, No. 4, 2002, pp. 259–272.

[3] Marzocca, P., Librescu, L., and Silva, W. A., "Aeroelastic Response of Swept Aircraft Wings in a Compressible Flow Field," AIAA Paper 2001-0714, Jan. 2001.

[4] Marzocca, P., Librescu, L., Kim, D.-H., and Lee, I., "Linear/Nonlinear Unsteady Aerodynamic Modeling of 2-D Lifting Surfaces via a Combined CFD/Analytical Approach," AIAA Paper 2003-1925, April 2003.

[5] Reisenthel, P. H., "Development of a Nonlinear Indicial Model for Maneuvering Fighter Aircraft," AIAA Paper 96-0896, Jan. 1996.

[6] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Dover, New York, 1996.

[7] Kim, D.-H., Lee, I., Marzocca, P., and Librescu, L., "Aeroelasticity of 2-D Lifting Surface via a Combined CFD/Analytical Approach," AIAA Paper 2004-1756, April 2004.

[8] Kim, D. H., and Lee, I., "Transonic and Low-Supersonic Aeroelastic Analysis of Two Degree of Freedom Airfoil with Freeplay Non-linearity," *Journal of Sound and Vibration*, Vol. 234, No. 5, 2000, pp. 859–880.

[9] Kim, D. H., Park, Y. M., Lee, I., and Kwon, O. J., "Nonlinear Aeroelastic Computation of a Wing/Pylon/Finned-Store Using Parallel Computing," *AIAA Journal*, Vol. 43, No. 1, 2005, pp. 53–62.